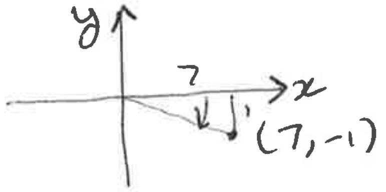
12. $\frac{2.2}{7-i} = z_1$

$|z_1| = \sqrt{50} = 5\sqrt{2}$

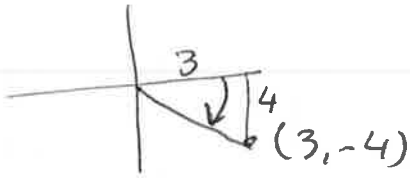


$\arg(z_1) = -\tan^{-1}\left(\frac{1}{7}\right)$

$z_2 = 3 - 4i$

$|z_2| = \sqrt{25} = 5$

$\mathrm{Arg}(z_2) = -\tan^{-1}\left(\frac{4}{3}\right)$



$$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|} = \frac{5\sqrt{2}}{5} = \sqrt{2}$$

$$\arg\left(\frac{z_1}{z_2}\right) = \arg z_1 - \arg z_2 = -\tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{4}{3}\right) = \tan^{-1}\left(\frac{4}{3}\right) - \tan^{-1}\left(\frac{1}{7}\right)$$

$$\therefore \boxed{\frac{z_1}{z_2} = \sqrt{2}\ \mathrm{Cis}\left[\tan^{-1}\left(\frac{4}{3}\right) - \tan^{-1}\left(\frac{1}{7}\right)\right)}$$

Actually, this step was not needed yet. The first part just requires $\left|\frac{z_1}{z_2}\right|$, which is $\sqrt{2}$.

O.K. Now evaluate $\tan\left\{\tan^{-1}\left(\frac{4}{3}\right) - \tan^{-1}\left(\frac{1}{7}\right)\right\}$

Well, let $\alpha^c = \tan^{-1}\left(\frac{4}{3}\right)$; $\theta^c = \tan^{-1}\left(\frac{1}{7}\right)$

Recall, $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

$$\therefore \tan(\alpha - \theta) = \frac{\tan\alpha - \tan\theta}{1 + \tan\alpha\tan\theta}$$

$$\Rightarrow \tan(\alpha - \theta) = \frac{\tan\tan^{-1}\left(\frac{4}{3}\right) - \tan\tan^{-1}\left(\frac{1}{7}\right)}{1 + \tan\tan^{-1}\left(\frac{4}{3}\right) \times \tan\tan^{-1}\left(\frac{1}{7}\right)}$$

$$= \frac{\frac{4}{3} - \frac{1}{7}}{1 + \frac{4}{3} \times \frac{1}{7}} = \frac{\frac{25}{21}}{\frac{25}{21}} = 1$$

We know $\arg\left(\frac{z_1}{z_2}\right) = \tan^{-1}\left(\frac{4}{3}\right) - \tan^{-1}\left(\frac{1}{7}\right)$

And we know $\arg\left(\frac{z_1}{z_2}\right)$ is an angle.

lets call it β.

$\therefore \quad \beta = \tan^{-1}\left(\frac{4}{3}\right) - \tan^{-1}\left(\frac{1}{7}\right)$

$\therefore \tan\beta = \tan\left[\tan^{-1}\left(\frac{4}{3}\right) - \tan^{-1}\left(\frac{1}{7}\right)\right]$

$\therefore \tan\beta = 1$

which quadrant is β in ?

$$\frac{7-i}{3-4i} \times \frac{3+4i}{3+4i} = \frac{25+25i}{25} = 1+i$$

$\therefore \beta$ is in 1st quadrant

$\Rightarrow \beta = \frac{\pi}{4}$

2.2

15. (a) $x^2+2x+3=0$

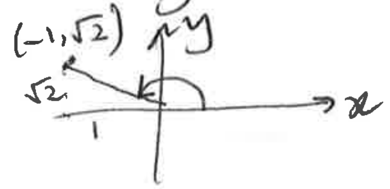
$$x = \frac{-2 \pm \sqrt{-8}}{2} = \frac{-2 \pm 2\sqrt{2}\,i}{2}$$

$$x = -1 \pm i\sqrt{2}$$

$$x = -1 + i\sqrt{2} \ ; \ x = -1 - i\sqrt{2}$$

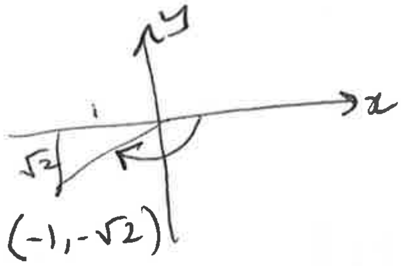
$|z_1| = \sqrt{3}$

$\arg z_1 = \pi - \tan^{-1}\sqrt{2}$

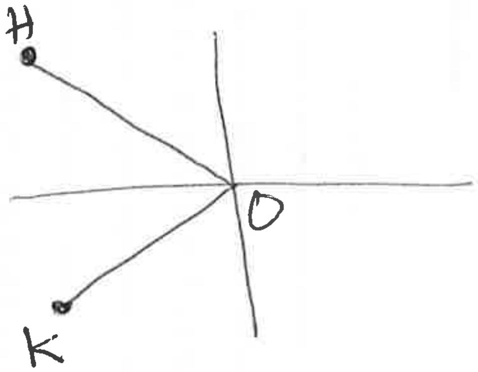


$|z_2| = \sqrt{3}$

$\arg z_2 = -\pi + \tan^{-1}\sqrt{2}$



(b)



$x^2+2px+q=0$

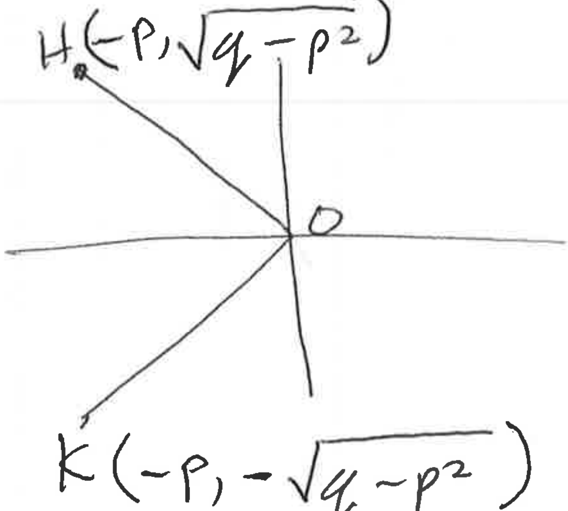
$$x = \frac{-2p \pm \sqrt{(2p)^2-4q}}{2} = \frac{-2p \pm \sqrt{4p^2-4q}}{2}$$

$$= \frac{-2p \pm \sqrt{-4(q-p^2)}}{2} = \frac{-2p \pm 2i\sqrt{q-p^2}}{2}$$

2.2

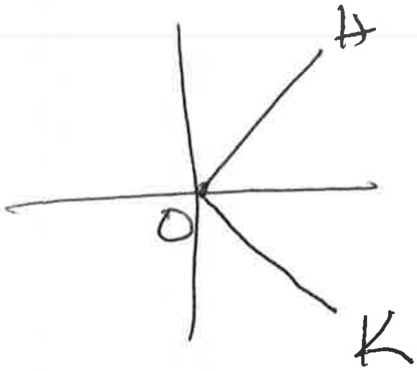
15(b) $x = -p \pm i\sqrt{q-p^2}$ since $p^2 < q$

$x = -p + i\sqrt{q-p^2}$; $x = -p - i\sqrt{q-p^2}$



$\boxed{p > 0}$

or



$\boxed{p < 0}$

For $H\hat{O}K = \frac{\pi}{2}$, then $m_{OK} \times m_{OH} = -1$

$$\therefore \frac{p}{\sqrt{q-p^2}} \times \frac{p}{-\sqrt{q-p^2}} = -1$$

$$\frac{p^2}{-(q-p^2)} = -1$$

$$p^2 = (q-p^2)$$

$$p^2 = q - p^2$$

$\therefore 2p^2 = q$ is the condition.

15 (b) equidistant from O.

$\therefore |AO| = |BO| = |HO| = |KO| = \sqrt{3}$

Since $|AO| = |BO| = \sqrt{3}$ from (a).

$\therefore |HO| = \sqrt{p^2 + q - p^2}$

$= \sqrt{q}$

$|KO| = \sqrt{p^2 + q - p^2}$

$= \sqrt{q}$

$\therefore \sqrt{q} = \sqrt{3}$

$q = 3$